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METHOD FOR CALCULATING CONVECTIVE DRYING OF MOIST MATERIALS

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The dependence of output of a drying apparatus in terms of evaporated moisture is obtained as a function of time and parameters and properties of the material being dried. Calculation results are compared with experiment.

The basic quantity which characterizes operation of drying apparatus is the quantity of water evaporated, which is usually determined from material balance equations. A method which would permit determination of this quantity from known or easily measured characteristics of the process and relate the engineering calculation of drying equipment output to the kinetics of drying is of practical interest.

We will write the combined equation of thermal balance and heat exchange for the operating conditions of a convective drying apparatus

$$[\alpha_{\text{cr}}(T_{\text{h}}-T_{\text{w}})+\bar{\alpha}(T_{\text{h}}-T_{\text{s}})]F\tau = rm_0N\tau_{\text{I}}+rm_0\frac{d\bar{u}}{d\tau}\tau_{\text{II}}+(c_0m_0+c_{\text{m}}m_{\text{m}}^{\text{II}})\frac{dT_{\text{mt}}}{d\tau}\tau_{\text{II}}. \tag{1}$$

In accordance with the definition of the Stenton number, this parameter in the first and second drying periods can be written in the following manner:

$$\text{St}_{\text{cr}}=\frac{\alpha_{\text{cr}}}{c_pv\rho}=\frac{T_1-T_2}{T_{\text{c}}-T_{\text{w}}}\frac{f}{F};\quad \overline{\text{St}}=\frac{\bar{\alpha}}{c_pv\rho}=\frac{T_1-T_2}{T_{\text{h}}-T_{\text{s}}}\frac{f}{F}. \tag{2}$$

With consideration of Eq. (2), we write Eq. (1) in the form

$$\left(1+\frac{\overline{\text{St}}}{\text{St}_{\text{cr}}}\frac{T_{\text{h}}-T_{\text{s}}}{T_{\text{h}}-T_{\text{w}}}\right)\tau=\frac{1}{c_pv\rho f(T_1-T_2)}\left[rm_0N\tau_{\text{I}}+rm_0\frac{d\bar{u}}{d\tau}\tau_{\text{II}}+(c_0m_0+c_{\text{m}}m_{\text{m}}^{\text{II}})\frac{dT_{\text{mt}}}{d\tau}\tau_{\text{II}}\right]. \tag{3}$$

According to [1], for convective drying

$$\frac{\overline{\text{Nu}}}{\text{Nu}_{\text{cr}}}\equiv\frac{\overline{\text{St}}}{\text{St}_{\text{cr}}}=(1+\text{Rb})N^{*0,57};\quad \frac{T_{\text{h}}-T_{\text{s}}}{T_{\text{h}}-T_{\text{w}}}=N^{*0,43}. \tag{4}$$

The drying rate in the first period can be expressed as

$$N=\frac{m_{\text{m}}^{\text{I}}}{m_0\tau_{\text{I}}}. \tag{5}$$

From the definition of the Rebinder number Rb, it follows that

$$\frac{dT_{\text{mt}}}{d\tau}=\frac{d\bar{u}}{d\tau}\frac{r}{c}\text{Rb}. \tag{6}$$

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With consideration of Eqs. (4)-(6), Eq. (3) takes on the form

$$[1+(1+\mathrm{Rb})N^*]\tau=\frac{1}{c_pM_{\mathrm{air}}\ (T_1-T_2)}\left[\left[rm^{\mathrm{I}}_{\mathrm{m}}+\frac{d\bar{u}}{d\tau}\tau_{\mathrm{II}}\right]\left[rm_0+(c_0m_0+c_{\mathrm{m}}m^{\mathrm{II}}_{\mathrm{m}})\frac{r}{c}\mathrm{Rb}\right]\right]. \tag{7}$$

According to [2], the drying rate in the second period can be expressed in terms of the drying rate in the first period:

$$\left|\frac{d\bar{u}}{d\tau}\right|=\varkappa N(\bar{u}-u_{\mathrm{p}}), \tag{8}$$

while the relative drying rate

$$N^*=\frac{\dfrac{d\bar{u}}{d\tau}}{N}=\varkappa(\bar{u}-u_{\mathrm{p}}). \tag{9}$$

We introduce the notation

$$\bar{u}-u_{\mathrm{p}}=\frac{m^{\mathrm{II}}_{\mathrm{m}}}{m_0};\quad n=\frac{m^{\mathrm{I}}_{\mathrm{m}}}{m_{\mathrm{m}}};\quad (1-n)=\frac{m^{\mathrm{II}}_{\mathrm{m}}}{m_{\mathrm{m}}}. \tag{10}$$

The quantity n, which is the ratio of the mass of moisture evaporated in the first period to the total moisture evaporated during the drying time depends on the curve of the process, and is thus a function of the form in which moisture is bound to the material.

With consideration of Eq. (10) the drying rate in the second period and the relative drying rate can be written in the form

$$\left|\frac{d\bar{u}}{d\tau}\right|=\varkappa n(1-n)\left(\frac{m_{\mathrm{m}}}{m_0}\right)^2\frac{1}{\tau_{\mathrm{I}}}, \tag{11}$$

$$N^*=\varkappa(1-n)\frac{m_{\mathrm{m}}}{m_0}. \tag{12}$$

Substituting Eqs. (11), (12) in Eq. (7), we obtain

$$\left[1+\varkappa(1-n)(1+\mathrm{Rb})\frac{m_{\mathrm{m}}}{m_0}\right]\tau=\frac{1}{c_pM_{\mathrm{air}}\ (T_1-T_2)}\left[\left[nrm_{\mathrm{m}}+\varkappa n(1-n)\left(\frac{m_{\mathrm{m}}}{m_0}\right)^2\frac{\tau_{\mathrm{II}}}{\tau_{\mathrm{I}}}\right]\left[rm_0+(c_0m_0+c_{\mathrm{m}})(1-n)\frac{m_{\mathrm{m}}}{m_0}\frac{r}{c}\mathrm{Rb}\right]\right]. \tag{13}$$

We now express the quantities τ_{I} and τ_{II} in terms of the total drying time τ. To do this, we write the simultaneous equation of heat exchange and thermal balance for the first drying period

$$\alpha_{\mathrm{cr}}(T_{\mathrm{h}}-T_{\mathrm{w}})F=rm_0N. \tag{14}$$

After transformations similar to those performed on Eq. (1), we have

$$\alpha_{\mathrm{cr}}(T_{\mathrm{h}}-T_{\mathrm{w}})F=r\frac{m^{\mathrm{I}}_{\mathrm{m}}}{\tau_{\mathrm{I}}}, \tag{15}$$

$$\mathrm{St}_{\mathrm{cr}}(T_{\mathrm{h}}-T_{\mathrm{w}})F=\frac{rnm_{\mathrm{m}}}{c_pv\rho\tau_{\mathrm{I}}}, \tag{16}$$

whence

$$\tau_{\mathrm{I}}=\frac{rnm_{\mathrm{m}}}{c_pM_{\mathrm{air}}\ (T_1-T_2)}. \tag{17}$$

The time for the second drying period can be expressed as

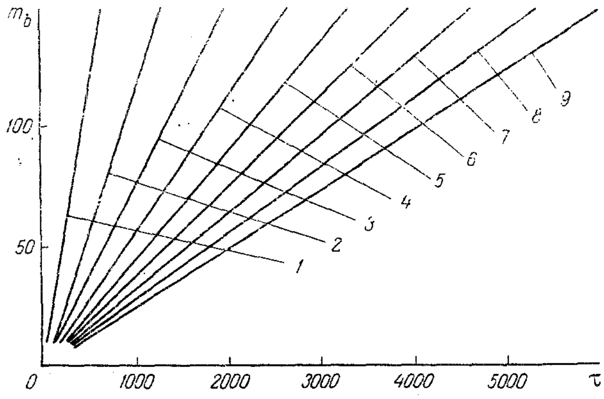

Fig. 1. Drying apparatus output m_m (kg) vs drying time τ (sec) and quantity n for m_0 = 100 kg and w_1 = 100%, M_{air} = 1 kg/sec and $\Delta T = T_1 - T_2$ = 50°K: 1) n = 0.1; 2) 0.2; 3) 0.3; 4) 0.4; 5) 0.5; 6) 0.6; 7) 0.7; 8) 0.8; 0.9.

$$\tau_{II}=\tau-\tau_I=\tau-\frac{rnm_m}{c_pM_{air}\,(T_1-T_2)}. \tag{18}$$

At the end of the process, when the moisture content becomes equal to its equilibrium value, $c_{\bar{u}=u_p}=c_0+c_m u_p$ and $\mathrm{Rb}_{\bar{u}=u_p}=\mathrm{const}$.

Substituting Eqs. (17), (18) in Eq. (13), we obtain

$$\left[1+\varkappa(1-n)(1+\mathrm{Rb}_{\bar{u}=u_p})\frac{m_m}{m_0}\right]\tau=\frac{1}{c_pM_{air}\,(T_1-T_2)}\left[\left[nrm_m+\frac{\varkappa n(1-n)m_m^2c_pM_{air}\,(T_1-T_2)}{nrm_0^2m_m}\left(\tau-\frac{rnm_m}{c_pM_{air}\,(T_1-T_2)}\right)\right][rm_0+(c_0m_0+c_m(1-n)\,m_m)]\frac{r}{c_{\bar{u}=u_p}}\mathrm{Rb}_{\bar{u}=u_p}\right]. \tag{19}$$

Equation (19) describes the process of convective drying of moist materials and establishes the dependence of drying apparatus production in terms of evaporated moisture on drying time, basic kinetic characteristics of the process, properties of the material being dried, and the form in which water is bound to the material, as well as the dependence on such important process parameters as mass flow rate and temperature of the heat-exchange agent. In this case the drying process extends from an initial material moisture w_1 to the equilibrium value w_e.

We will transform Eq. (19) to form convenient for numerical solution. We take c_0 = 1400 J/(kg·deg); c_m = 4220; c_p = 1005 (the heat-exchange agent is hot air); $c_{\bar{u}=u_p}$ = 1780 J/(kg·deg) (for capillary porous bodies at u_p = 0.08); $r = 2250\cdot10^3$ J/kg. With consideration of these parameter values characterizing the drying process, after corresponding transformations Eq. (19) can be represented by a third-order polynomial

$$pm_m^3+\beta m_m^2+\gamma m_m+K\tau=0, \tag{20}$$

where

$$p=CBKE=215\cdot10^8\frac{n^2(1-n)^2\,\mathrm{Rb}_{\bar{u}=u_p}}{w_1m_0M_{air}\,(T_1-T_2)}, \tag{21}$$

$$\beta=C(BKD-E\tau)=9067\cdot10^6\frac{n^2(1-n)(1+0{,}79\,\mathrm{Rb}_{\bar{u}=u_p})}{w_1m_0M_{air}\,(T_1-T_2)}-$$

$$-963\cdot10^3\frac{n(1-n)^2}{w_1m_0^2}\mathrm{Rb}_{\bar{u}=u_p}\tau,$$

$$\gamma=KA\tau-BK^2-CD\tau=(KA-CD)\tau-BK^2= \tag{22}$$

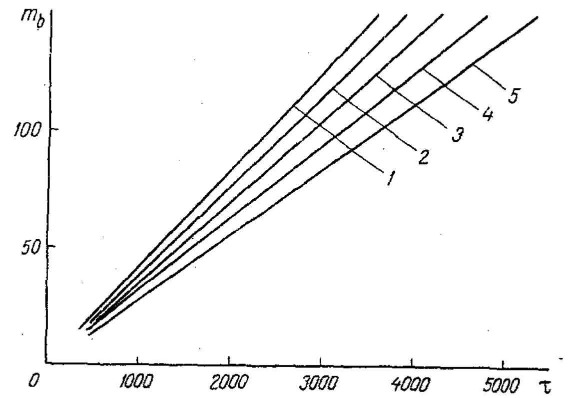

Fig. 2. Drying apparatus output m_m vs drying time τ and mass flow rate of heat-exchange agent M_{air} (kg/sec) at m_0 = 100 kg, w_1 = 100%, $\Delta T = T_1 - T_2 = 70$°K and n = 0.9; 1) M_{air} = 1.2; 2) 1.1; 3) 1.0; 4) 0.9; 5) 0.8.

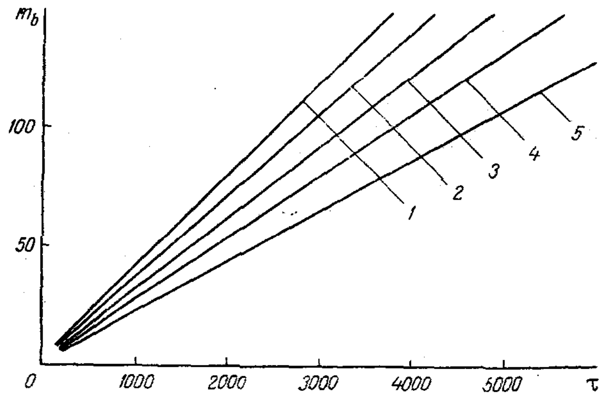

Fig. 3. Drying apparatus output m_m vs drying time τ and temperature heat $\Delta T = T_1 - T_2$ (°K) at m_0 = 100 kg, w_1 = 100%; M_{air} = 1 kg/sec and n = 0.9: 1) ΔT = 90; 2) 80; 3) 70; 4) 60; 5) 50.

$$= \frac{850\cdot 10^3 n(1-n)}{w_1 m_0}\tau - 5037\cdot 10^6 \frac{n^2}{M_{air}\ (T_1 - T_2)}, \tag{23}$$

$$A = \frac{\varkappa(1-n)(1+\mathrm{Rb}_{\bar{u}=u_p})}{m_0} = \frac{1{,}8(1-n)(1+\mathrm{Rb}_{\bar{u}=u_p})}{w_1 m_0}, \tag{24}$$

$$B = \frac{1}{c_p M_{air}\ (T_1 - T_2)} = \frac{1}{1005 M_{air}\ (T_1 - T_2)}, \tag{25}$$

$$C = \frac{\varkappa n(1-n)}{m_0^2} = \frac{1{,}8n(1-n)}{w_1 m_0^2}, \tag{26}$$

$$D = rm_0 + c_0 m_0 \frac{r}{c_{\bar{u}=u_p}} \mathrm{Rb}_{\bar{u}=u_p} = 2250\cdot 10^3 m_0 (1 + 0{,}79\,\mathrm{Rb}_{\bar{u}=u_p}), \tag{27}$$

$$E = c_m(1-n)\frac{r}{c_{\bar{u}=u_p}}\mathrm{Rb}_{\bar{u}=u_p} = 5335\cdot 10^3 (1-n)\,\mathrm{Rb}_{\bar{u}=u_p},$$

$$K = nr = 2250\cdot 10^3 n.$$

TABLE 1. Comparison of Results of Numerical (Eq. (20)) and Experimental Determinations of Ouptut of "Eliteks" Dryer

Drying time τ, sec	Drying apparatus output m_m, kg		Drying time τ, sec	Drying apparatus output m_m, kg	
	numerical sol.	expt.		numerical sol.	expt.
520	8,1	9,3	720	9,8	10,1
600	8,9	10,4	780	11,4	11,3
650	9,6	10,5	830	12,5	13,2

Equation (20) was solved numerically using a computer for convective drying conditions for various moist materials over a wide range of kinetic and process characteristics. The effect of various factors on output was evaluated, and the form of the material being dried was modeled by change in the value n over the range 0.1...0.9, corresponding to a wide spectrum of capillary porous bodies.

The calculations performed show that the mathematical description of the process is realized stably for a dry mass of material in the range m_0 = 10...600 kg with variation of initial moisture level w_1 from 80 to 150%.

It was found that the Rebinder criterion, which was varied over the range $Rb_{\bar{u}=u_p}$ = 0.9, has practically no effect on the output of the drying apparatus. The latter depends significantly on the character of the drying process and the form of the material being dried, i.e., on the value n. Thus, for operation of a dryer with M_{air} = 1 kg/sec and $T_1 - T_2$ = 50°K, with variation of n from 0.9 to 0.3 the quantity m_m increases by a factor of three (Fig. 1). This is explained by the fact that with decrease in n the drying process shifts into the region of the period with decreasing rate, where rapid penetration of the evaporation zone into the material and intensification of heat–mass exchange processes occur.

The output of the drying apparatus in terms of evaporated moisture increases in proportion to increase in the mass flow rate (velocity of motion) of the heat-exchange agent (Fig. 2) and increase in temperature in the drying chamber (Fig. 3).

The numerical solutions obtained were compared with results of an experimental study of the process of fabric drying in an "Eliteks" commercial dryer. Conditions were M_{air} = 0.86 kg/sec, $\Delta T = T_1 - T_2$ = 80°K, m_0 = 10 kg, w_1 = 90%, and n = 0.9. Results of the comparison with consideration of the actual efficiency of the dryer are presented in Table 1, from which the good agreement between experiment and the numerical solutions can be seen.

NOTATION

α_{cr}, α, heat liberation coefficients in first and second periods, W/(m$^2\cdot$K); T_h, T_w, T_s, temperatures of heat-exchange agent, wet bulb thermometer, and material surface, K; F, surface area of material, m^2; τ_I, τ_{II}, $\tau=\tau_I+\tau_{II}$, durations of first and second drying periods and total drying time, sec; m_0, m_m^I, m_m^{II}, $m_m=m_m^I+m_m^{II}$, masses of dry material, moisture, moisture evaporated moisture (drying apparatus output), kg; N, $d\bar{u}/d\tau$, drying rate in first and second periods, sec^{-1}; $dT_{st}/d\tau$, rate of change of material temperature in second period, K/sec; r, latent heat of evaporation, J/kg; c_0, c, c_m, specific heats of absolutely dry material, moist material, and moisture, J/(kg·K); $M_{air}=v\rho f$, c_p, v, ρ, mass flow rate of heat-exchange agent, kg/sec, specific heat, J/(kg·sec), velocity, m/sec, and density, kg/m^3; f, cross-sectional area for heat-exchange agent flow, m^2; T_1, T_2, heat-exchange agent temperatures at input and output of drying chamber; °K; κ, relative drying coefficient, 1%; $\bar{u}$, u_p, current and equilibrium material moisture content.

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